SECTION A: ANSWERS TO THE COMPANY'S QUESTIONS

PART I

a) Which is the minimal price per MWh the Company could offer to its three prospective customers such that all costs for buying, transporting, storing and balancing gas are covered? Which contracts should the Company enter into?

Answer:

i. The total minimum cost of transporting gas to all three customers is £6,534,236.828 at a total demand of 164,844. Therefore the minimal price per MWh the company could offer the customers is

$$Minimal\ Price\ per\ MWh = \left[\frac{164{,}844}{164{,}844}\right] = \textbf{£39.64}$$

ii. The Company should enter into Flat Band Contract 01/10, Flat Band Contract 21/30, Flat Band Contract 01/15, Flat Band Contract 01/30 and a Swing Contract.

b) Is the solution to your model unique? If no, interpret the solution and explain in terms of the gas marketing problem (not in mathematical terms) why this is not the case.

Answer: NO. The solution to the model is not unique. With the nature of problem the company faces yet with a lot of possible and different scenarios with the data given, there are several other alternate options in gas transport paths, stocking and balancing. For instance on every day, some gas is injected through contracts but, there is also availability of supply at the stock or in the balance sheet. Using this kind of option or the other can result in the same total cost for the company.

c) Is the optimal combination of contracts that the model in a) has found (be it transport contracts, flat band contracts, the swing contract, the storage contract or any combination of these) unique or is it possible that a transport via other pipelines or buying gas on the basis of other types of contract, for

example, could also lead to an optimal solution? To determine the answer, use a mathematical model derived from the one you used in a).

Since our model is not a unique solution, it is possible to obtain different combination of variables that lead to a solution of the same value. Therefore we can model a Mixed Integer model taking the current solution and forcing it to achieve a new solution.

d). The Company has the opportunity to bid on an auction for an additional amount of contracted swing capacity. Which is the maximum the company should be willing to pay for an additional capacity of 1 MWh/day?

Answer: The solution to our model costs: **£6,534,236.83**.

With this cost, an increase in swing contract capacity of 1MWh/day will reduce our cost to **£6,534,235.83.** This means it is possible to pay up for this price to increase in the swing contract capacity.

e). The Company considers entering into negotiations with the consortium that owns pipeline P6. Which is the maximal amount per unit of contracted capacity (GBP/MWh/day) that the consortium should charge to make P6 interesting for the company?

Answer: The maximal amount per unit charge to make P6 interesting for the company is by reducing the usage charge of P6 from £784.5 to 767.59.

f) Customer 4 is hesitating to have gas delivered by the Company. If the Company could convince customer 4 to sign a contract under the same conditions that customers 1, 2 and 3 are willing to agree to, which price per MWh could the Company offer to all four customers? Please provide, on the basis of the model data, an intuitive explanation of the sign of the change of the price per MWh. (I.e. if the new price that results from delivering all 4 customers is higher than the price calculated in a), please explain why. If this price turns out to be lower than the price calculated in a), provide an intuitive explanation of why it is lower.)

Answer: The current minimum price is **£39.64**. Therefore by adding Customer 4's demand our cost is reduced to **£38.41.**

A statistical interpretation of this result is that, by adding customer 4, the standard deviation of demand per day is forced down from 124 to 60, which means that a more stable injection of gas through contracts can be delivered and therefore possible to avoid larger stock contracts and gas waste. We can also make use of cheaper contracts, such as Contract 1/30.

That is to say that in a typical buying on the market scenario, the more of a quantity of an item you buy the less the price is.

Part II

Having carried out a more detailed analysis of their pattern of gas consumption, customer 4 has arrived at the conclusion that it is wise to base their contract with our company on three demand scenarios, which are given in the table on page 7. The scenarios A, B and C are assumed to have a probability of 0.25, 0.5 and 0.25, respectively.

a) If the Company has already signed supply contracts with customers 1, 2 and 3 based on the price calculated in question a) in Part I, which price per MWh could it offer to customer 4? (The Company wishes to offer to customer 4 the minimal expected cost per MWh that is incurred by delivering gas to customer 4 in addition to delivering gas to customers 1, 2 and 3.)

Answer: From the results of Part I we observed that:

Total Cost	Total Delivery	Minimal Price
£6,534,236.828	164,844	£39.64

Adding the demand of customer 4, our estimated cost is **£8,343,747.568.**
This means that, our cost of supplying gas to customer 4 is **£1,809,510.74.**

The table below show the three scenarios with their total demand and cost of gas. There we can offer it to customer 4 at 40.11 [GBP/MWh].

Scenario	Demand	Cost/MWh
A	51,085 [MWh]	35.42 [GBP/MWh]
B	45,114[MWh]	**40.11 [GBP/MWh]**
C	39,126[MWh].	46.23[GBP/MWh]

b) The management of the Company is sceptical about using the scenarios provided by customer 4 and suggests basing the offer on the demand given in the table on page 1, which would result in a lower offer and hence increase the probability of doing business with customer 4. Comment on this suggestion in your report.

Answer: By using the expect demand of customer 4, our total cost is **£8,063,643.27**. Therefore our cost for adding customer 4, based of our model in Part I is:

£8,063,643.27 - £6,534,236.83 = £1,529,406.44

With Scenario B at £ 45,114 we get a price of **£33.90/MWh**

This is not good in terms of risk, because whilst Customers 1, 2 and 3 have accepted to bear all risks, customer 4 is not ready to bear any risk, therefore our plan could fail eventually. In other words if we use our stochastic plan, we are guaranteed a better response to variations.

Part III

In view of the new market conditions (i.e. the increased balancing capacity and the offer to use the storage in City B), the Company wishes to know if, when delivering gas to customers 1, 2 and 3 as in question a) in part I, it should sign a contract with the owner of the storage in City B. Adapt the model used for question a) in part I to this case.

Answer: Yes.
From our new model and its solution, the best option to will be to use the storage in City B. With this option, the cost is reduced to **£6,484,944.98.**

If we do not allow the use of storage B and still use the 20% of balancing capacity, our total cost is: **£6,506,545.10** moving our price offered to customers at **39.34 (GBP/MWh).**

SECTION A: BUILDING THE MODEL

We began by building the linear model to Part I. Based on this model, we extended it into a stochastic and integer models by introducing new Sets, Variables and Parameters to our linear model.

2.1 ASSUMPTIONS

- For modelling purposes we assumed that gas can be transported through a pipeline into both directions at the same time
- There are 30 working days in a month
- Flat Band Contracts and Swing Band Contracts are not mutually exclusive.

2.2 SETS

Cust	Set of all Customers
Days	Set of all Days in a Working Month (30 days)
Loc	Set of all Locations
FBC	Set of All Flat Band Contracts
Pipes	Set of all Pipes

2.3 INDICES

fb	index for the flat band contracts
d1, d2	indices for Days
l1, l2	indices for Location
p	index for pipeline

2.4 **PARAMETERS**

- Customers Daily Demand.
- Customer Location
- Days that Flat Band Contract delivers gas.
- Maximum Daily delivery of Flat Band Contract.
- Price paid for Flat Band Contract.
- Number of days in a Flat Band Contract
- Maximum Total gas delivered by use of swing contract.

- Total Cost of Swing Contract Gas
- Cost of daily use of swing contract.
- Pipeline connection between cities
- Maximum Pipeline capacity.
- Cost of contracted pipeline capacity.
- Cost for contracted pipeline volume use
- Stock Withdrawal Capacity Cost
- Stock Injection Capacity Cost
- Stock Capacity Cost

2.5 DECISION VARIABLES

- Quantity of gas delivered to customer each day
- Quantity of gas negotiated for flat a band contract
- Total gas contracted for Swing Contracts
- Quantity of Swing Contract gas used on a daily basis
- Daily cost due to use of Swing Contract
- Contracted capacity for Pipe during month
- Quantity of gas transported from one city to the other on a specific day.
- Maximum contracted stock
- Maximum Stock Injection
- Maximum Stock Withdrawal
- Total gas stocked at the end of day at Location 'City C'
- Balance Sheet and Balancing Capacity

2.6 CONSTRAINTS

The constraints captured for the model were analysed and developed. A set of all the constraints can be found in the Appendix A.

2.6.1The key constraints are

- Customer Demand must be met
- Pipeline Capacity
- Contracted Volume Capacity
- Inflow and Outflow constraints through the cities

2.6.2 Flat Band and Swing Contract Constraints

- Maximum Quantity for Flat Band Contract
- Maximum Daily Use for Swing Contract
- Minimal Daily Use for Swing Contract
- Daily Cost of Swing Contract
- Swing Contract Minimal Daily Cost
- Swing Contract Maximum Daily Cost

2.6.3 Stock Constraints

- Contracted Maximum Stock
- Maximum Stock Injection and Maximum Stock Withdrawal
- Maximum Daily Withdrawal from Contracted Stock
- Maximum Balance Capacity (if under 2000 MWh distribution capacity
- Maximum Balance Capacity (if over 2000 MWh distribution capacity)
- Maximum Daily injection Balance and Maximum Daily Withdraw Balance
- Maximum Overall Balance and Minimum Overall Balance

2.7 OBJECTIVE FUNCTION

The objective function here is to minimize the various costs associated with transporting gas to customers; these are contract, transportation and storage costs.

2.7.1 Contract Cost

- Contracted Flat Band Cost
- Total Swing Contract Cost
- Daily Swing Contract Usage Cost

2.7.2 Transportation Cost

- Contracted Capacity Cost
- Pipeline Usage Cost

2.7.3 Storage Cost

- Stock Capacity Cost
- Stock Injection Capacity Cost and Withdrawal Capacity Cost

SECTION C: RESULTS AND CONCLUSION

3.0 A Summary of Results from Linear Model

Flat Band	Contracted Amount	Upper Bound	Reduced Cost
Contract 01/10	412.64784	2000	0
Contract 11/20	0	2000	103.4
Contract 21/30	343.72296	2000	0
Contract 01/15	423.32435	10000	0
Contract 16/30	0	10000	53.85
Contract 01/30	5102.183	50000	0

Pipeline	Contracted Capacity
P1	6426.1283
P2	179.02194
P3	0
P4	758.58586
P5	122.11055
P6	0
P7	1,998
P8	4,364
P9	644.13779

Fixed Swing Contract Cost	2000
Total Swing Contract Cost	14080
Maximum Stock Inject	177.23172
Maximum Stock Withdrawal	14.353014
Maximum Contracted Stock	358.8253386
Balance Capacity	421.30641

Swing Contract Usage

Day	Daily Use	Stock	Balance Sheet	Days	Daily Use	Stock	Balance Sheet
Day1	66.66667	0	-421.306	Day16	40	177.2317	1263.919
Day2	40	0	-714.592	Day17	66.66667	162.8787	842.6128
Day3	40	0	-372.518	Day18	55.63672	148.5257	586.0588
Day4	66.66667	0	-488.676	Day19	40	134.1727	687.3148
Day5	40	0	-569.873	Day20	40	119.8197	685.603
Day6	66.66667	0	-468.65	Day21	66.66667	105.4667	798.3503
Day7	51.02994	0	-294.772	Day22	66.66667	91.11364	842.6128
Day8	40	0	-276.746	Day23	40	100.4711	1263.919
Day9	40	0	-81.4203	Day24	66.66667	86.11808	1102.495
Day10	66.66667	0	270.5446	Day25	66.66667	71.76507	870.2963
Day11	40	0	252.5342	Day26	66.66667	57.41205	770.6908
Day12	40	0	283.4776	Day27	66.66667	43.05904	669.2231
Day13	40	0	357.4137	Day28	66.66667	28.70603	497.5696
Day14	40	0	596.5976	Day29	66.66667	14.35301	137.9368
Day15	40	0	842.6128	Day30	66.66667	0	0

3.1 A Summary of Results from Stochastic Model

Flat Band	Contracted Amount	Upper Bound	Reduced Cost
Contract 01/10	0	2000	15.7
Contract 11/20	0	2000	87.7
Contract 21/30	39.59412937	2000	0
Contract 01/15	388.1015033	10000	0
Contract 16/30	0	10000	53.85
Contract 01/30	7100.0816885102.183	50000	0

Pipeline	Contracted Capacity
P1	7757.148703
P2	520.2588581
P3	0
P4	758.5858586
P5	265.2613065
P6	0
P7	1,855
P8	5,840
P9	2520.872761

Fixed Swing Contract Cost	2000
Maximum Stock Inject	515.05627
Maximum Stock Withdrawal	15.246077
Maximum Contracted Stock	11264.90425
Balance Capacity	487.85744

3.2 A Summary of Results from Integer Model

Flat Band	Contracted Amount	Upper Bound	Reduced Cost
Contract 01/10	279.52401	2000	0
Contract 11/20	0	2000	51.592
Contract 21/30	0	2000	51.808
Contract 01/15	0	10000	3.054
Contract 16/30	0	10000	50.796
Contract 01/30	5525.2979	50000	0

Pipeline	Contracted Capacity
P1	6426.1283
P2	0
P3	0
P4	758.58586
P5	122.11055
P6	0
P7	1,998
P8	4,364
P9	644.13779

4.0 CONCLUSION

In this project, we developed a mathematical model and used AMPL studio to find an optimum minimal cost of transporting gas to four customers of a gas marketing company.

The minimum costs derived are tabulated below.

Model Type	Customers	Total Minimum Cost
Linear	1, 2 and 3	£6,534,236.828
Stochastic	1, 2, 3 and 4	£8,3437,47.56
Integer	1, 2 and 3	£6,484,944.98

From our Linear Model we obtained the minimal price per MWh to the three customers at **£39.64**. By adding customer 4 with its expected demand, the minimal price went down to **£38.41**.